**Tracking**

**Simple Motion Model**

State vector:

Linear Motion Model with constant acceleration.

Write the motion model in state space representation

Output …..

In this project, is set to be 1 second for simplicity.

**Sensor Fusion**

Problem: If have a noisy laser and noisy radar sensor at the origin, can we track a moving car accurately by combining laser and radar measurements. This topic is important because in order to drive a car safely, we must be able to know the positions and velocities of other cars.

**Sensors**

1. Laser

Laser uses an infrared laser beam to determine the distance between the sensor and a nearby object. Laser cannot measure the velocity of objects directly and has to rely on two or more position measurements to estimate the velocity.

In practice, the measurement of a laser sensor is point clouds. For simplicity, here assume that the measurement from a laser sensor is already computed to the 2D location.

The state vector is

The measurement matrix of a laser sensor is as below.

Based on measurement matrix above, it can be found that laser sensors can only measure x position and y position. The x and y velocity can be calculated by state transition matrix.

The laser measurement noise covariance matrix is as below.

Here assume that x direction noise and y direction noise are uncorrelated.

The matrix R represents the uncertainty in the position measurements receive from the laser sensor.

2. Radar

Radar uses radio waves and Doppler effect to measure speed directly, thus it is important for sensor fusion because it helps fusion algorithms converge much faster.

Different from measurements from laser sensors which are in cartesian plane, measurements from radar sensors are in polar plane. The measurement vector is as below.

Where is the distance between a laser sensor and the observed object, is the angle between the ray and x axis, and is Doppler velocity which represents the velocity of the observed object.

The radar measurement noise covariance matrix is as below.

The measurement function:

Here assume that three measurement noises are uncorrelated.

**Kalman Filter**

Kalman Filter is the most popular technique for solving tracking problems because the it’s very efficient. In Kalman Filter, the distribution is given by a unimodal Gaussian. The task of Kalman Filter is to maintain mean(Mu) and variance(Sigma square) as the best estimate of the location.

Kalman Filter algorithm iterates on two cycles. The first one is the measurement update which is based on Bayes Rule. Taking a product of prior distribution and measurement probability and

then normalize it to get posterior distribution. The second is the predict process applying total probability. The pseudo code of time-varying-gain Kalman Filter is as below.

Prediction:

Measurement update:

H is measurement matrix

R is measurement noise

P is uncertainty covariance matrix

K is Kalman gain matrix

I is identity matrix

F is state transition matrix

Q is noise covariance

u is input

z is measurement

**Extended Kalman Filter**

The Kalman Filter algorithm works well for laser sensors because of the linearity of measurements from laser sensors. However, the measurement function of radar sensors is not a linear function, thus, during the measurement update process, the distribution becomes non-Gaussian. Therefore, Extended Kalman Filter is necessary to predict states from non-linear measurements.

Extended Kalman Filter can deal with non-linear model (non-linear state transition matrix) or non-linear measurement functions, the ideal of it is linearizing these matrices by first order Taylor expansion.

where is Jacobian matrix: Hj

Given that the motion model is a linear model, there is no need to linearize state transition matrix. The pseudo code of time-varying-gain Extended Kalman Filter is as below.

Prediction:

Measurement update:

**Localization**

After implementing Kalman Filter and Extended Kalman Filter, the location of the observed object can be known. Another important issue is that how to know our present location relative to fixed coordinate when we are tracking the target.